Differential Options Functioning by Multinomial Logistic Regression

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The focus of this study: differential options functioning

❖ **Differential Options Functioning (DOF)**

- Differential options functioning (DOF) is defined as an investigation of **whether different groups of respondents, with same levels or status on attribute being measured, would have different response to the *options***

- Statistically, DOF detects **group difference in *probability of choosing each option* compared to the reference option** after controlling for the attribute being measured

The focus of this study: differential options functioning

❖ **Differential Options Functioning (DOF)**

Q. What is 2 x (1+3) – 1 ?

A. 3 → incorrect
B. 6 → incorrect
C. 4 → Incorrect
D. 7 → **Answer (Reference Option)**

- DOF investigates the group difference in the following way:

A vs. D → $P(A)_{group1} = P(A)_{group2}$?

B vs. D → $P(B)_{group1} = P(B)_{group2}$?

C vs. D → $P(C)_{group1} = P(C)_{group2}$?

The purpose of this study

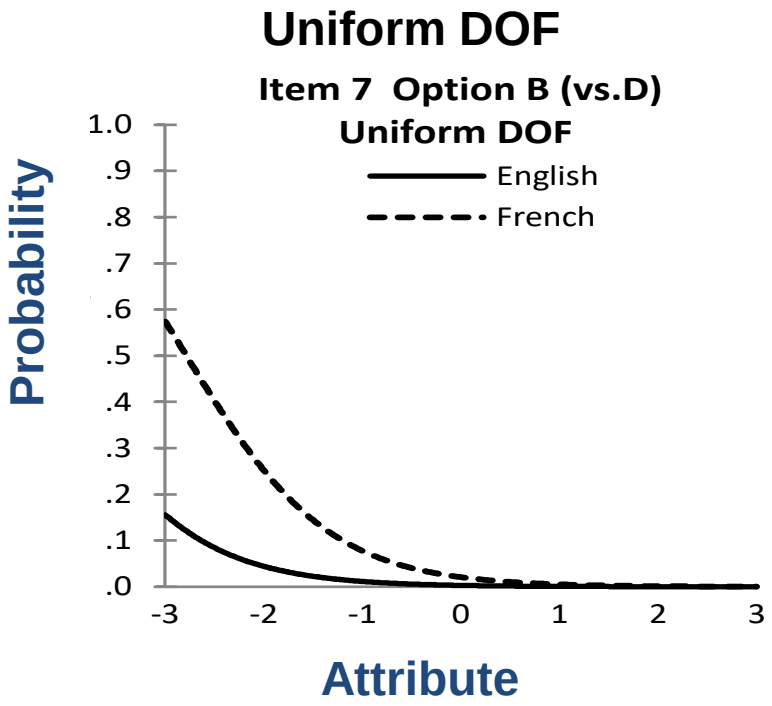
❖ **This study aims to fulfill the following purposes:**

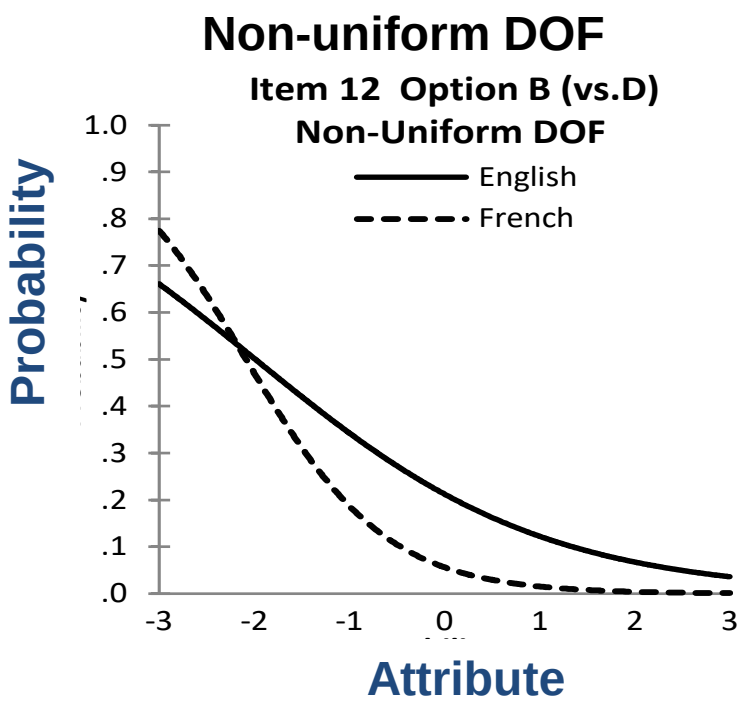
- Propose a simple and integrated analytical method for DOF based on multinomial logistic regression

- Re-conceptualize previous approach to DOF (referred to as differential distractor functioning, DDF) in terms of its terminologies, purposes, and uses

The proposed analytical method for DOF

- This method is based on **multinomial logistic regression**
- Two types of DOF: **uniform DOF** and **non-uniform DOF**





The proposed analytical method for DOF

❖ Three multinomial logistic regression models

$$\text{Model} - 1: \; log \frac{P(Y=j \mid T)}{P(Y=k \mid T)} = a_j + b_1 \boldsymbol{T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\text{Model} - 2: \; log \frac{P(Y=j \mid T,G)}{P(Y=k \mid T,G)} = a_j + b_1 \boldsymbol{T} + b_2 \boldsymbol{G} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\text{Model} - 3: \; log \frac{P(Y=j \mid T,G)}{P(Y=k \mid T,G)} = a_j + b_1 \boldsymbol{T} + b_2 \boldsymbol{G} + b_3 (\boldsymbol{T} * \boldsymbol{G}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$j = 1 \ldots$ J denotes the categories of the available options,

k denotes the base (reference) category (e.g., a keyed option),

T is the rest total score,

G is the grouping variable,

$T * G$ is the interaction between the two variables.

The proposed analytical method for DOF

❖ **The procedures consists of two stages**

- **Stage 1:**

Whether the item has ***at least*** **one option showing DOF** based on likelihood ratio test (LRT)

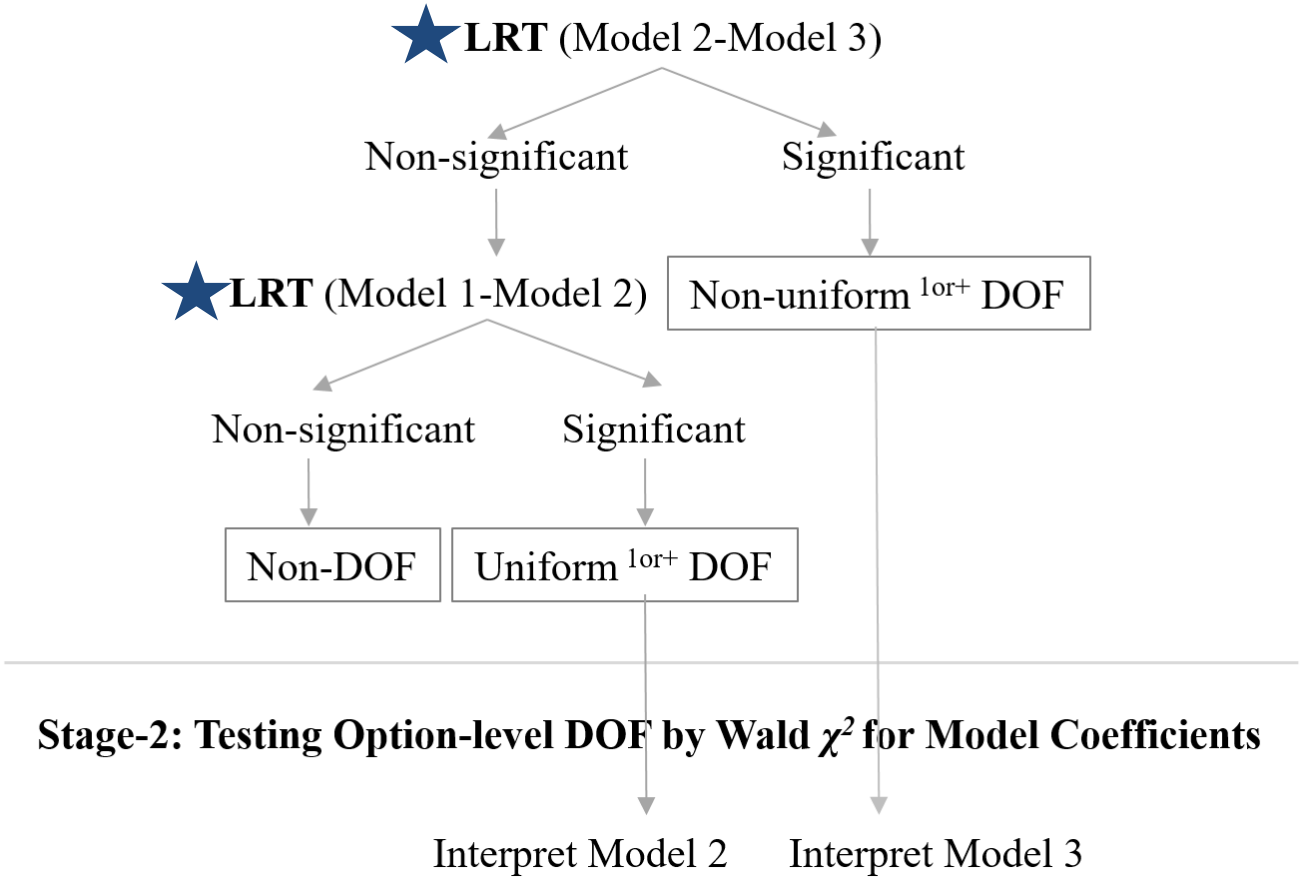
- **Stage 2:**

Which **options show DOF and what type of DOF** based on Wald χ^2 test

The proposed analytical method for DOF

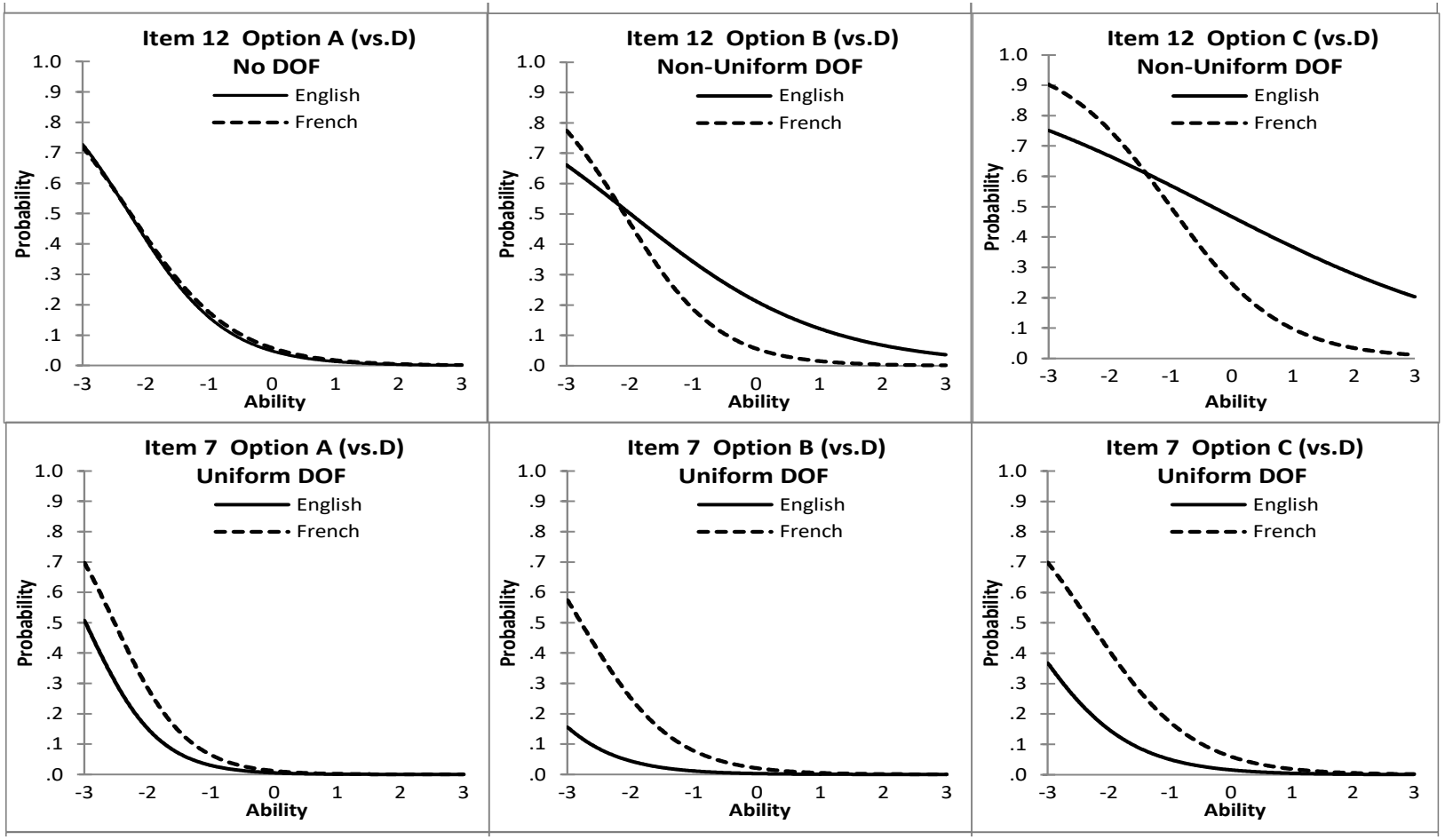
❖ The procedures consists of two stages

Stage-1: Testing Item-level Overall DOF by Likelihood Ratio Test (LRT)



The proposed analytical method for DOF

- Option Characteristic Curves (OCCs)



Potential uses of DOF

❖ **DOF expands its uses to investigating practical issues**

❖ **Potential uses of DOF in the two contexts:**

- Achievement/Aptitude tests
- Questionnaires/Surveys

Potential uses of DOF : achievement/aptitude tests

❖ **Achievement/aptitude tests**

- Options are written to represent possible misunderstanding, lack of knowledge, or missteps to reach the correct answer

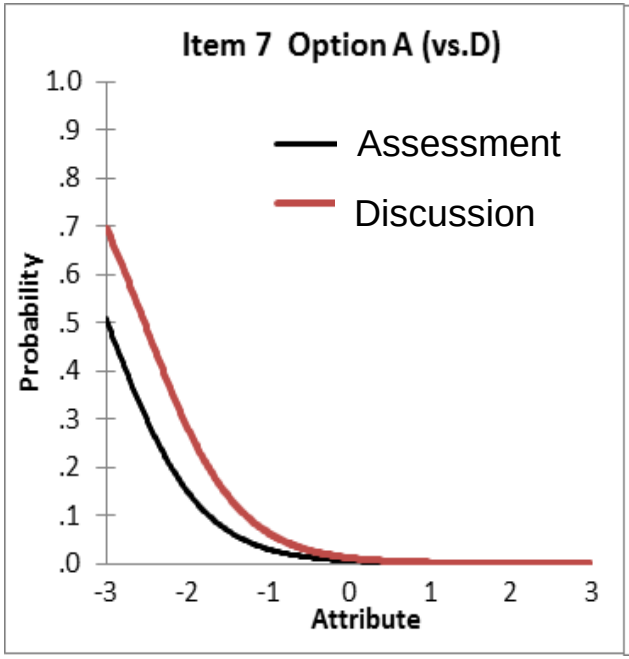
- DOF can be applied to understand how the groups make mistakes in answering the item (accounting for ability)

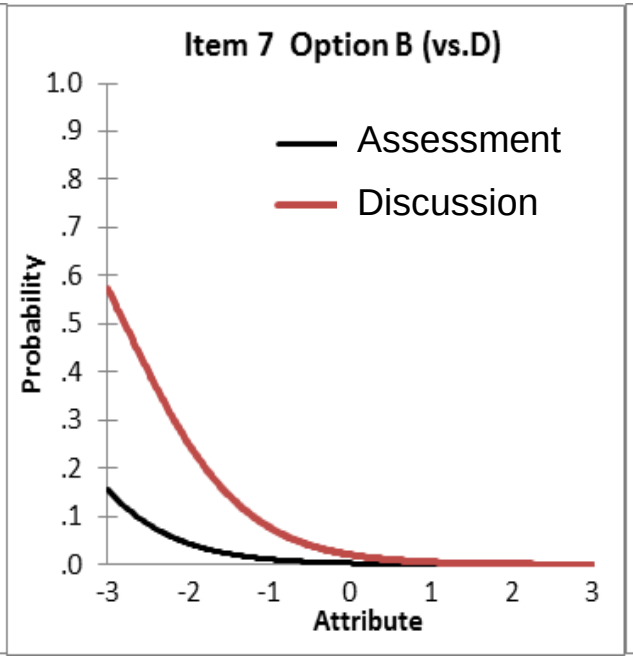
- The information of non-keyed option is useful to see which concepts are difficult for the groups

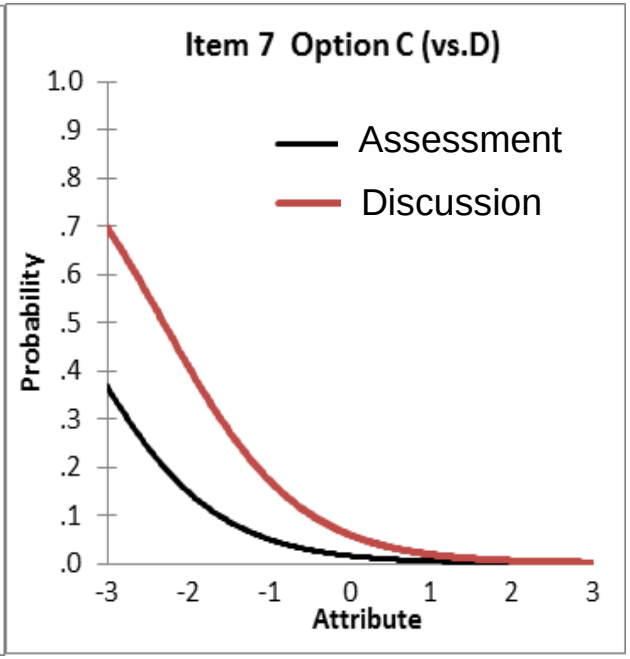
Potential uses of DOF: achievement/aptitude tests

❖ **Hypothetical example**

- Assessment method vs. Discussion method







Note. D= correct answer (reference option), A, B, C = incorrect answer

Note. Attribute is ability being measured in cognitive tests

Potential uses of DOF: questionnaires/surveys

❖ **Questionnaires/Surveys (No correct answer)**

- Options are written to represent different levels of endorsement or different types of choices

  e.g. “Strongly disagree”, “Disagree”, “Neutral”, “Agree”, “Strongly agree”

  e.g. “Vanilla”, “Strawberry”, “Chocolate”

- DOF can provide **information of response patterns of the groups** of individuals

- DOF can show how the groups prefer or avoid options differently

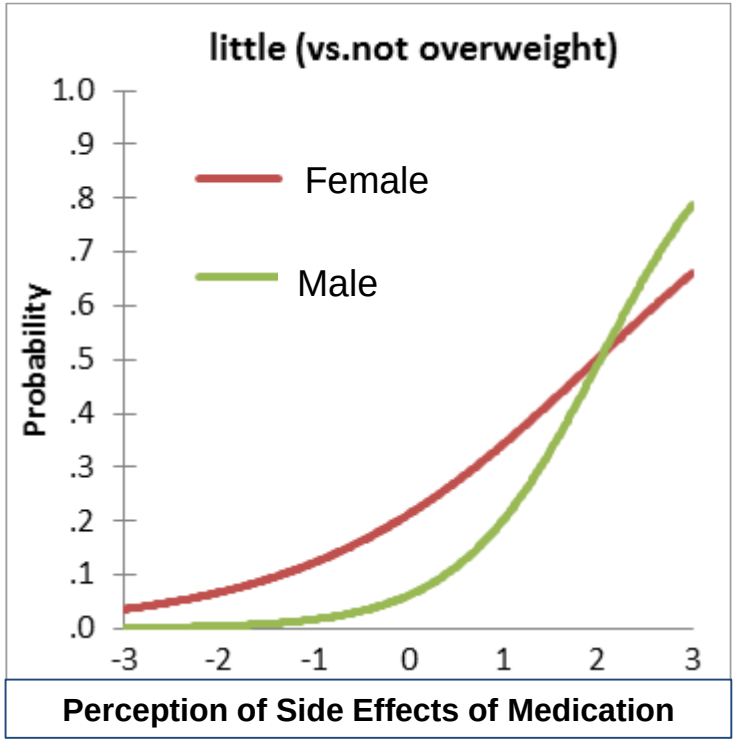
Potential uses of DOF: questionnaires/surveys

❖ Hypothetical example

Perception of body weights
(Montejo et al., 2011)

- ○ I am not overweight ★
- ○ I am a little overweight
- ○ I am rather much overweight
- ○ I am very overweight

★= reference option

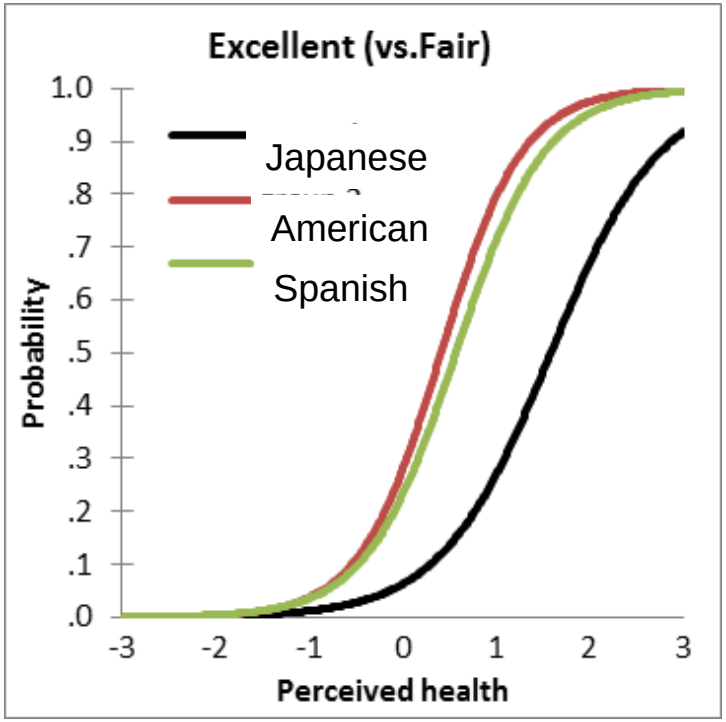


Potential uses of DOF: questionnaires/surveys

❖ Hypothetical example

In general, would you say that your health is…

- ○ Excellent
- ○ Very good
- ○ Good
- ○ Fair ★
- ○ Poor



★ = reference option

Relationship between DIF and DOF

❖ **Statement 1:** *If DIF occurs, then DOF occurs* → True

Probability of choosing an option conditioning on ability (trait)			Population DIF/DOF status
Options	Group 1	Group 2	
A (key)	0.25	0.40	DIF (favoring group 2)
B	0.25	0.25	No DOF
C	0.30	0.30	No DOF
D	0.20	0.15	DOF (favoring group 1)
	1.00	1.00	

❖ **Statement 2:** *If DOF occurs, then DIF occurs* → False

Probability of choosing an option conditioning on ability (trait)			Population DIF/DOF status
Options	Group 1	Group 2	
A (key)	.25	.25	No DIF
B	.30	.25	DOF (favoring group 1)
C	.25	.40	DOF (favoring group 2)
D	.20	.10	DOF (favoring group 1)
	1.00	1.00	

Relationship between DIF and DOF

❖ **Statement 3:** *If DOF does not occur, then DIF does not occur*

Probability of choosing an option conditioning on ability (trait)			Population DIF/DOF status
Options	Group 1	Group 2	
A (key)	0.45	0.45	No DIF
B	0.20	0.20	No DOF
C	0.10	0.10	No DOF
D	0.25	0.25	No DOF
	1.00	1.00	

❖ **Statement 4:** *If DIF does not occur, then DOF does not occur*

Probability of choosing an option conditioning on ability (trait)			Population DIF/DOF status
Options	Group 1	Group 2	
A (key)	0.25	0.25	No DIF
B	0.30	0.50	DOF (favoring group 2)
C	0.25	0.10	DOF (favoring group 1)
D	0.20	0.15	DOF (favoring group 1)
	1.00	1.00	

Relationship between DIF and DOF

❖ **Inference from the statements**

- The existence of DIF indicates the existence of DOF (statement-1)
- The existence of DOF is not a sign of DIF (statement-2)
- The absence of DOF indicates the absence of DIF (statement-3)
- The absence of DIF does not indicate that there is no DOF (statement-4)

Contribution of this study

- This study expands the application of DOF to investigating practical issues

- The new method makes the study of DOF accessible in practice
  - Can be conducted easily in popular statistical packages (e.g., R, Mplus, SAS, STATA)
  - Applied to psychological and educational measurement contexts
  - Investigate more than two groups simultaneously
  - Model multiple options simultaneously